Regularization of Large Scale Total Least Squares Problems

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Many problems in data estimation are governed by overdetermined linear systems

$$Ax \approx b, \quad A \in \mathbb{R}^{m \times n}, \ b \in \mathbb{R}^m, m \geq n. \tag{1}$$

In the classical least squares approach the system matrix A is assumed to be free from errors, and all errors are confined to the observation vector b. However, in engineering application this assumption is often unrealistic. For example, if the matrix A is an approximation to the true operator or if not only the right-hand side b but A as well are obtained by measurements, then both are contaminated by some noise.

An appropriate approach to this problem is the total least squares (TLS) method which determines perturbations $\Delta A \in \mathbb{R}^{m \times n}$ to the coefficient matrix and $\Delta b \in \mathbb{R}^m$ to the vector b such that

$$\|[\Delta A, \Delta b]\|_F^2 = \min! \quad \text{subject to } (A + \Delta A)x = b + \Delta b, \tag{2}$$

where $\| \cdot \|_F$ denotes the Frobenius norm of a matrix.

In practical situations the linear system is often ill-conditioned. Then the TLS method often yields yields physically meaningless solutions, and regularization is necessary to stabilize the computed solution.

In this presentation we discuss two approaches for regularizing large scale TLS problems. One which is based on adding a quadratic constraint to (2) through which the size of the solution is bounded or a certain degree of smoothness is imposed [1,2], and a Tikhonov type regularization concept which penalizes excessive growth or erratic behavior of the solution [3].

References

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